Derivatives Leftovers

The notebook contains the code not included in the Derivatives Books:

It is raw code and contains a few errors. Some of the graphs differ from the graphs on the website. I have no present intention to turn this into an easy-to-use package, but everyone should feel free to use and and adapt it under the CC Share-Alike license.

```
GT := 3
```

Definition of Derivative

A real valued function f may have a derivative, which is another real valued function denoted by f'. At a point a, f'[a] is the slope of the tangent line to the curve y = f[x] at the point (a, f[x]).

Functions used in creating the examples

```
ShowDerivatives[f_, n_, a_, b_] :=
 Plot[Take[{f[x], f'[x], f''[x], f'''[x], f''''[x]}, n] // Evaluate, {x, a, b},
  Prolog → AbsoluteThickness[GT], PlotRange -> {a, b}, AspectRatio -> 1,
  ImageSize → {200, 200}, PlotStyle -> {{RGBColor[0, 0, 1]}, {RGBColor[1, 0, 0]},
    {RGBColor[0, 1, 0]}, RGBColor[.7, .7, 0], RGBColor[.7, 0, .7]}]
```

TO DO : Make SD print each line in the appropriate color.

TO DO : Increase the type size in the SD printout. It is not obvious how to do this.

TO DO : Bring the graph of the function to the front so that the derivatives all pass underneath it.

```
SD[f_, n_, a_, b_, c_, d_, s_] := ((r := (d - c) / (b - a));
  Plot[Take[{f[x], f'[x], f''[x], f'''[x], f''''[x], f'''''[x]}, n] // Evaluate,
   {x, a, b}, PlotRange -> {c, d}, Prolog → AbsoluteThickness[GT],
   AspectRatio → r, ImageSize → { s, r s},
   PlotStyle -> {{RGBColor[0, 0, 1]}, {RGBColor[1, 0, 0]}, {RGBColor[0, 1, 0]},
     RGBColor[.7, .7, 0], RGBColor[.7, 0, .7], RGBColor[0, .7, .7]}])
```

The derivatives are shown in each graph by color.
function: blue
first derivative : red
second : green
third : gold

fourth : purple
fifth: light blue

DF prints out the formulas for the derivatives

```
DF[f_, n_] :=
 TraditionalForm[
  TableForm[
   Take[{{"function", f[x]}, {"1st deriv", f'[x]},
     {"2nd deriv", f''[x]}, {"3rd deriv", f'''[x]}, {"4th deriv", f''''[x]},
     {"5th deriv", f'''''[x]}, {"6th deriv", f''''''[x]}}, n]]]
```

A function that is its own third derivative

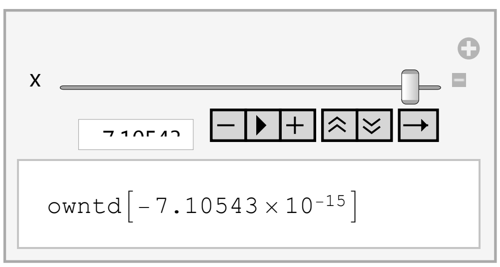
TO DO : This function has very complicated behavior left of 0 that I don' t understand because it takes on extreme values that I cannot plot.

```
DSolve[u'''[x] == u[x], u[x], x]
```

$$\left\{\left\{u[x] \to e^{x}\, C[1] + e^{-x/2}\, C[2]\, \mathrm{Cos}\left[\frac{\sqrt{3}\, x}{2}\right] + e^{-x/2}\, C[3]\, \mathrm{Sin}\left[\frac{\sqrt{3}\, x}{2}\right]\right\}\right\}$$

$$\texttt{owntd[x\_] := } e^{x} + e^{-x/2}\ \mathrm{Cos}\left[\frac{\sqrt{3}\ x}{2}\right]$$

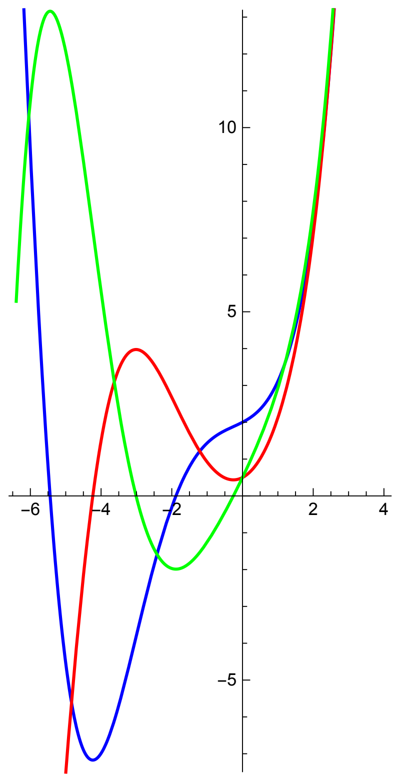
```
Manipulate[owntd[x], {x, -50, 2, Appearance → "Open"}] // N
```



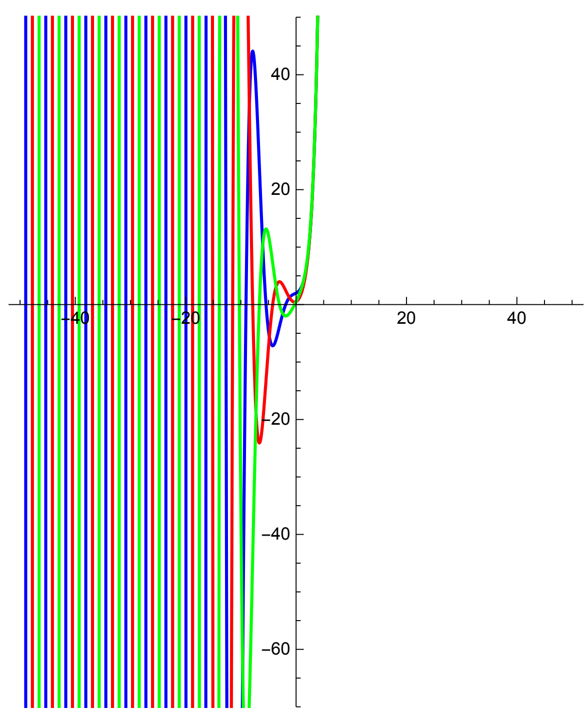

```
DF[owntd, 4]
```

function	$e^{x} + e^{-x/2} \cos\left(\frac{\sqrt{3}\,x}{2}\right)$
1st deriv	$e^{x} - \frac{1}{2}\sqrt{3}\; e^{-x/2} \sin\left(\frac{\sqrt{3}\,x}{2}\right) - \frac{1}{2} e^{-x/2} \cos\left(\frac{\sqrt{3}\,x}{2}\right)$
2nd deriv	$e^{x} + \frac{1}{2}\sqrt{3}\; e^{-x/2} \sin\left(\frac{\sqrt{3}\,x}{2}\right) - \frac{1}{2} e^{-x/2} \cos\left(\frac{\sqrt{3}\,x}{2}\right)$
3rd deriv	$e^{x} + e^{-x/2} \cos\left(\frac{\sqrt{3}\,x}{2}\right)$

```
SD[owntd, 3, -6.4, 4, -7.5, 13.2, 200]
```




```
SD[owntd, 3, -50, 50, -70, 50, 300]
```



A function that is its own third derivative

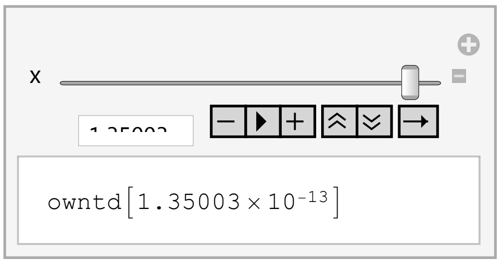
TO DO : This function has very complicated behavior left of 0 that I don' t understand because it takes on extreme values that I cannot plot.

```
DSolve[u'''[x] == u[x], u[x], x]
```

$$\left\{\left\{u[x] \to e^{x} C[1] + e^{-x/2} C[2] \operatorname{Cos}\left[\frac{\sqrt{3}\, x}{2}\right] + e^{-x/2} C[3] \operatorname{Sin}\left[\frac{\sqrt{3}\, x}{2}\right]\right\}\right\}$$

$$\text{owntd}[x\_] := e^{x} + e^{-x/2} \operatorname{Cos}\left[\frac{\sqrt{3}\, x}{2}\right]$$

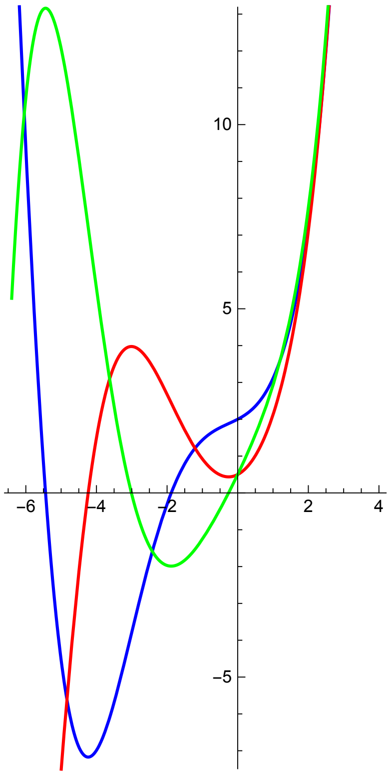
```
Manipulate[owntd[x], {x, -50, 2, Appearance → "Open"}] // N
```



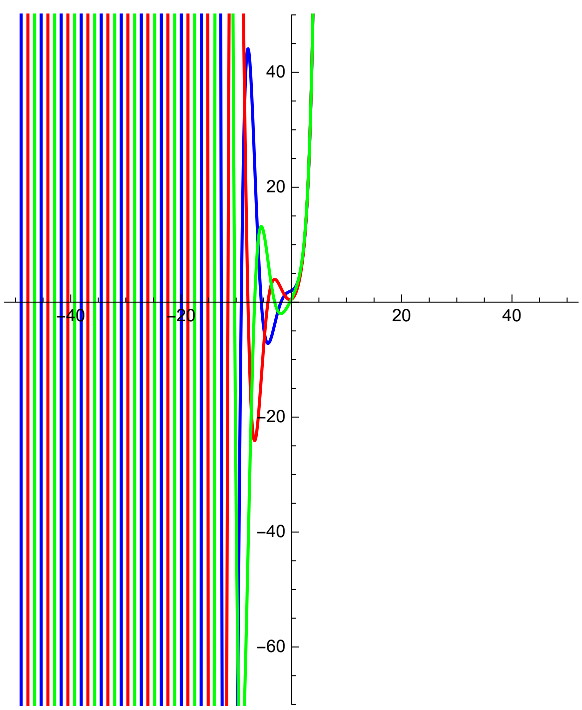

```
DF[owntd, 4]
```

function	$e^{x} + e^{-x/2} \cos\left(\frac{\sqrt{3}\, x}{2}\right)$
1st deriv	$e^{x} - \frac{1}{2}\sqrt{3}\, e^{-x/2} \sin\left(\frac{\sqrt{3}\, x}{2}\right) - \frac{1}{2} e^{-x/2} \cos\left(\frac{\sqrt{3}\, x}{2}\right)$
2nd deriv	$e^{x} + \frac{1}{2}\sqrt{3}\, e^{-x/2} \sin\left(\frac{\sqrt{3}\, x}{2}\right) - \frac{1}{2} e^{-x/2} \cos\left(\frac{\sqrt{3}\, x}{2}\right)$
3rd deriv	$e^{x} + e^{-x/2} \cos\left(\frac{\sqrt{3}\, x}{2}\right)$

```
SD[owntd, 3, -6.4, 4, -7.5, 13.2, 200]
```




```
SD[owntd, 3, -50, 50, -70, 50, 300]
```



A function that is its own fifth derivative

I have not been able to make these work. TO DO : Try later.

Sine and Cosine are functions that are their own fourth derivatives

```
DSolve[u'''''[x] == u[x], u[x], x]
```

$$\left\{\left\{u[x]\to e^x C[1]+e^{\left(-\frac{1}{4}-\frac{\sqrt{5}}{4}\right)x}C[3]\,\text{Cos}\left[\sqrt{\frac{5}{8}-\frac{\sqrt{5}}{8}}\,x\right]+e^{\left(-\frac{1}{4}+\frac{\sqrt{5}}{4}\right)x}C[2]\,\text{Cos}\left[\sqrt{\frac{5}{8}+\frac{\sqrt{5}}{8}}\,x\right]+e^{\left(-\frac{1}{4}-\frac{\sqrt{5}}{4}\right)x}C[4]\,\text{Sin}\left[\sqrt{\frac{5}{8}-\frac{\sqrt{5}}{8}}\,x\right]+e^{\left(-\frac{1}{4}+\frac{\sqrt{5}}{4}\right)x}C[5]\,\text{Sin}\left[\sqrt{\frac{5}{8}+\frac{\sqrt{5}}{8}}\,x\right]\right\}\right\}$$

```
ownfd[x_] := %
```

$$\text{ownfd}[x\_] := e^{\left(-\frac{1}{4}-\frac{\sqrt{5}}{4}\right)}\ \text{Cos}\left[\sqrt{\frac{5}{8}-\frac{\sqrt{5}}{8}}\,x\right]+e^{\left(-\frac{1}{4}+\frac{\sqrt{5}}{4}\right)}\ \text{Cos}\left[\sqrt{\frac{5}{8}+\frac{\sqrt{5}}{8}}\,x\right]$$

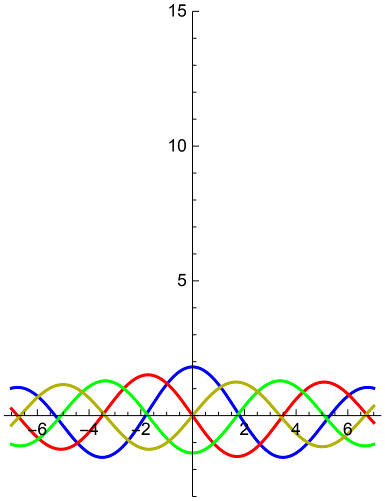
```
DF[ownfd, 6]
```

function	$e^{-\frac{1}{4}-\frac{\sqrt{5}}{4}}\cos\left(\sqrt{\frac{5}{8}-\frac{\sqrt{5}}{8}}\,x\right)+e^{\frac{\sqrt{5}}{4}-\frac{1}{4}}\cos\left(\sqrt{\frac{5}{8}+\frac{\sqrt{5}}{8}}\,x\right)$
1st deriv	$-\sqrt{\frac{5}{8}-\frac{\sqrt{5}}{8}}\,e^{-\frac{1}{4}-\frac{\sqrt{5}}{4}}\sin\left(\sqrt{\frac{5}{8}-\frac{\sqrt{5}}{8}}\,x\right)-\sqrt{\frac{5}{8}+\frac{\sqrt{5}}{8}}\,e^{\frac{\sqrt{5}}{4}-\frac{1}{4}}\sin\left(\sqrt{\frac{5}{8}+\frac{\sqrt{5}}{8}}\,x\right)$
2nd deriv	$-\left(\frac{5}{8}-\frac{\sqrt{5}}{8}\right)e^{-\frac{1}{4}-\frac{\sqrt{5}}{4}}\cos\left(\sqrt{\frac{5}{8}-\frac{\sqrt{5}}{8}}\,x\right)-\left(\frac{5}{8}+\frac{\sqrt{5}}{8}\right)e^{\frac{\sqrt{5}}{4}-\frac{1}{4}}\cos\left(\sqrt{\frac{5}{8}+\frac{\sqrt{5}}{8}}\,x\right)$
3rd deriv	$\left(\frac{5}{8}-\frac{\sqrt{5}}{8}\right)^{3/2}e^{-\frac{1}{4}-\frac{\sqrt{5}}{4}}\sin\left(\sqrt{\frac{5}{8}-\frac{\sqrt{5}}{8}}\,x\right)+\left(\frac{5}{8}+\frac{\sqrt{5}}{8}\right)^{3/2}e^{\frac{\sqrt{5}}{4}-\frac{1}{4}}\sin\left(\sqrt{\frac{5}{8}+\frac{\sqrt{5}}{8}}\,x\right)$
4th deriv	$\left(\frac{5}{8}-\frac{\sqrt{5}}{8}\right)^{2}e^{-\frac{1}{4}-\frac{\sqrt{5}}{4}}\cos\left(\sqrt{\frac{5}{8}-\frac{\sqrt{5}}{8}}\,x\right)+\left(\frac{5}{8}+\frac{\sqrt{5}}{8}\right)^{2}e^{\frac{\sqrt{5}}{4}-\frac{1}{4}}\cos\left(\sqrt{\frac{5}{8}+\frac{\sqrt{5}}{8}}\,x\right)$
5th deriv	$-\left(\frac{5}{8}-\frac{\sqrt{5}}{8}\right)^{5/2}e^{-\frac{1}{4}-\frac{\sqrt{5}}{4}}\sin\left(\sqrt{\frac{5}{8}-\frac{\sqrt{5}}{8}}\,x\right)-\left(\frac{5}{8}+\frac{\sqrt{5}}{8}\right)^{5/2}e^{\frac{\sqrt{5}}{4}-\frac{1}{4}}\sin\left(\sqrt{\frac{5}{8}+\frac{\sqrt{5}}{8}}\,x\right)$

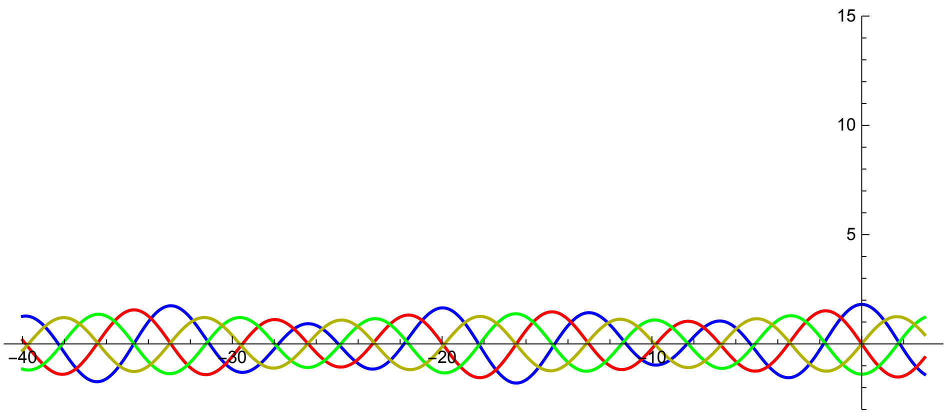
```
ownfd[-20] // N
```

```
1.6489
```

```
SD[ownfd, 4, -7, 7, -3, 15, 200]
```




```
SD[ownfd, 4, -40, 3, -3, 15, 500]
```




```
Remove[ownfd]
```

A function that is its own fifth derivative second try

Sine and Cosine are functions that are their own fourth derivatives

```
DSolve[u'''''[x] == u[x], u[x], x] // InputForm
```

```
{{u[x] -> E^x*C[1] + E^((-1/4 - Sqrt[5]/4)*x)*C[3]*Cos[Sqrt[5/8 - Sqrt[5]/8]*x] +
    E^((-1/4 + Sqrt[5]/4)*x)*C[2]*Cos[Sqrt[5/8 + Sqrt[5]/8]*x] + E^((-1/4 - Sqrt[5],
     Sin[Sqrt[5/8 - Sqrt[5]/8]*x] + E^((-1/4 + Sqrt[5]/4)*x)*C[5]*Sin[Sqrt[5/8 + Sq
```

```
ownfd[x_] := .2 E^x + E^((-1/4 - Sqrt[5] / 4) * x) * Cos[Sqrt[5/8 - Sqrt[5] / 8] * x] +
  E^((-1/4 + Sqrt[5] / 4) * x) * Cos[Sqrt[5/8 + Sqrt[5] / 8] * x] +
  E^((-1/4 - Sqrt[5] / 4) * x) * Sin[Sqrt[5/8 - Sqrt[5] / 8] * x] +
  E^((-1/4 + Sqrt[5] / 4) * x) * Sin[Sqrt[5/8 + Sqrt[5] / 8] * x]
```

ownfd[x]

$$0.2\, e^{x} + e^{\left(-\frac{1}{4}-\frac{\sqrt{5}}{4}\right) x} \operatorname{Cos}\left[\sqrt{\frac{5}{8}-\frac{\sqrt{5}}{8}}\; x\right] + e^{\left(-\frac{1}{4}+\frac{\sqrt{5}}{4}\right) x} \operatorname{Cos}\left[\sqrt{\frac{5}{8}+\frac{\sqrt{5}}{8}}\; x\right] + e^{\left(-\frac{1}{4}-\frac{\sqrt{5}}{4}\right) x} \operatorname{Sin}\left[\sqrt{\frac{5}{8}-\frac{\sqrt{5}}{8}}\; x\right] + e^{\left(-\frac{1}{4}+\frac{\sqrt{5}}{4}\right) x} \operatorname{Sin}\left[\sqrt{\frac{5}{8}+\frac{\sqrt{5}}{8}}\; x\right]$$

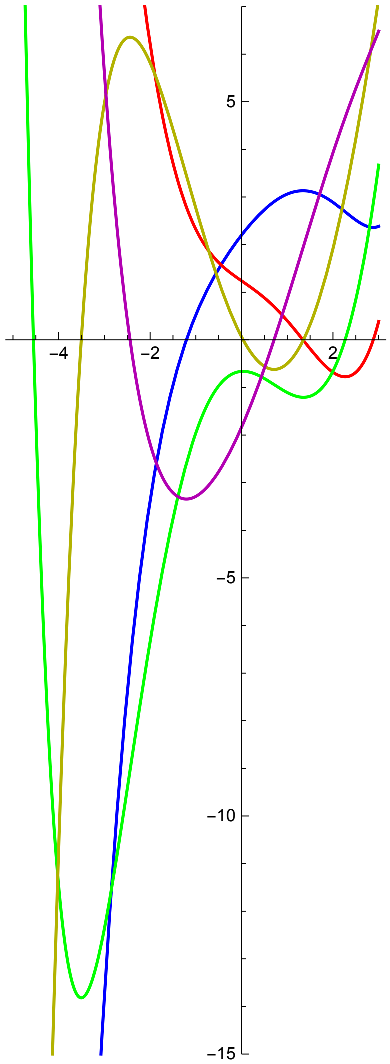
ownfd[-20] // N

1.50456×10^{7}

DF[ownfd, 6]

function	$0.2\, e^{x} + e^{\left(-\frac{1}{4}-\frac{\sqrt{5}}{4}\right)x} \sin\left(\sqrt{\frac{5}{8}-\frac{\sqrt{5}}{8}}\; x\right) + e^{\left(\frac{\sqrt{5}}{4}-\frac{1}{4}\right)x} \sin\left(\sqrt{\frac{5}{8}+\frac{\sqrt{5}}{8}}\; x\right) + e^{\left(-\frac{1}{4}-\frac{\sqrt{5}}{4}\right)x} \cos\left(\sqrt{\frac{5}{8}-\frac{\sqrt{5}}{8}}\; x\right) + e$
1st deriv	$0.2\, e^{x} - \sqrt{\frac{5}{8}-\frac{\sqrt{5}}{8}}\; e^{\left(-\frac{1}{4}-\frac{\sqrt{5}}{4}\right)x} \sin\left(\sqrt{\frac{5}{8}-\frac{\sqrt{5}}{8}}\; x\right) + \left(-\frac{1}{4}-\frac{\sqrt{5}}{4}\right) e^{\left(-\frac{1}{4}-\frac{\sqrt{5}}{4}\right)x} \sin\left(\sqrt{\frac{5}{8}-\frac{\sqrt{5}}{8}}\; x\right) + \left(\frac{\sqrt{5}}{4}-\frac{1}{4}\right)$
2nd deriv	$0.2\, e^{x} - \left(\frac{5}{8}-\frac{\sqrt{5}}{8}\right) e^{\left(-\frac{1}{4}-\frac{\sqrt{5}}{4}\right)x} \sin\left(\sqrt{\frac{5}{8}-\frac{\sqrt{5}}{8}}\; x\right) - 2\left(-\frac{1}{4}-\frac{\sqrt{5}}{4}\right)\sqrt{\frac{5}{8}-\frac{\sqrt{5}}{8}}\; e^{\left(-\frac{1}{4}-\frac{\sqrt{5}}{4}\right)x} \sin\left(\sqrt{\frac{5}{8}-\frac{\sqrt{5}}{8}}\; x\right)$
3rd deriv	$-\left(\frac{5}{8}-\frac{\sqrt{5}}{8}\right)^{3/2} e^{\left(-\frac{1}{4}-\frac{\sqrt{5}}{4}\right)x} \cos\left(\sqrt{\frac{5}{8}-\frac{\sqrt{5}}{8}}\; x\right) - 3\left(-\frac{1}{4}-\frac{\sqrt{5}}{4}\right)\left(\frac{5}{8}-\frac{\sqrt{5}}{8}\right) e^{\left(-\frac{1}{4}-\frac{\sqrt{5}}{4}\right)x} \cos\left(\sqrt{\frac{5}{8}-\frac{\sqrt{5}}{8}}\; x\right) + 3\,($
4th deriv	$\left(\frac{5}{8}-\frac{\sqrt{5}}{8}\right)^{2} e^{\left(-\frac{1}{4}-\frac{\sqrt{5}}{4}\right)x} \cos\left(\sqrt{\frac{5}{8}-\frac{\sqrt{5}}{8}}\; x\right) - 4\left(-\frac{1}{4}-\frac{\sqrt{5}}{4}\right)\left(\frac{5}{8}-\frac{\sqrt{5}}{8}\right)^{3/2} e^{\left(-\frac{1}{4}-\frac{\sqrt{5}}{4}\right)x} \cos\left(\sqrt{\frac{5}{8}-\frac{\sqrt{5}}{8}}\; x\right) - 6\,(-$
5th deriv	$\left(\frac{5}{8}-\frac{\sqrt{5}}{8}\right)^{5/2} e^{\left(-\frac{1}{4}-\frac{\sqrt{5}}{4}\right)x} \cos\left(\sqrt{\frac{5}{8}-\frac{\sqrt{5}}{8}}\; x\right) + 5\left(-\frac{1}{4}-\frac{\sqrt{5}}{4}\right)\left(\frac{5}{8}-\frac{\sqrt{5}}{8}\right)^{2} e^{\left(-\frac{1}{4}-\frac{\sqrt{5}}{4}\right)x} \cos\left(\sqrt{\frac{5}{8}-\frac{\sqrt{5}}{8}}\; x\right) - 10\,($

```
SD[ownfd, 5, -5, 3, -15, 7, 200]
```

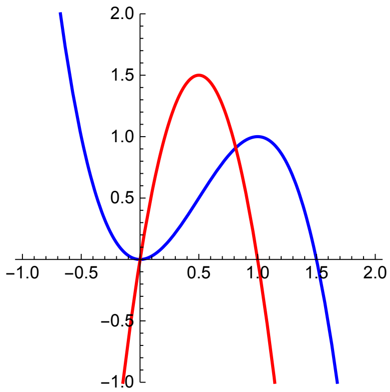



```
]
```

Beta

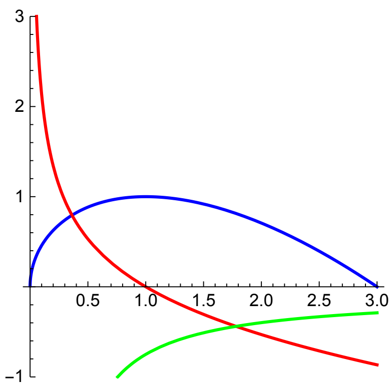
```
f[x_] := BetaRegularized[x, 2, 2]
```

```
ShowDerivatives[f, 2, -1, 2]
```




```
f[x_] := BetaRegularized[x, .5, 2]
```

```
ShowDerivatives[f, 3, -1, 3]
```

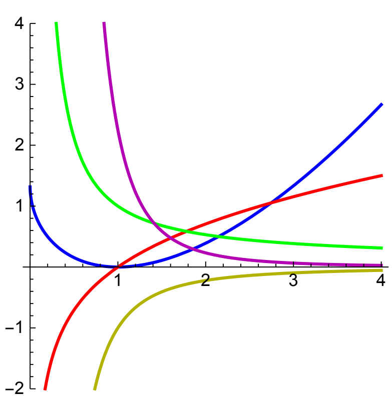



```
f[x_] := Beta[x, 1, .5, 2]
```

```
f[x]
```

```
Beta[x, 1, 0.5, 2]
```

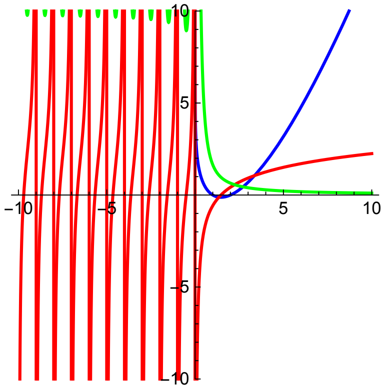
```
ShowDerivatives[f, 5, -2, 4]
```



LogGamma

```
f[x_] := LogGamma[x]
```

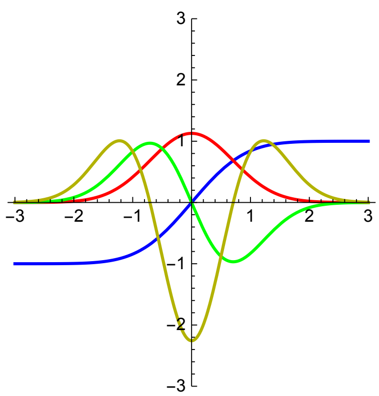
```
ShowDerivatives[f, 3, -10, 10]
```



Erf

```
f[x_] := Erf[x]
```

```
ShowDerivatives[f, 4, -3, 3]
```



Fresnel

```
g[x_] := Integrate[f[x], x]
```

```
g[x]
```

$$-\sqrt{\frac{\pi}{2}}\left(-\mathrm{Cos}[1]\ \mathrm{FresnelS}\left[\sqrt{\frac{2}{\pi}}\ x\right]+\mathrm{FresnelC}\left[\sqrt{\frac{2}{\pi}}\ x\right]\ \mathrm{Sin}[1]\right)$$

g[1]

Integrate::ilim : Invalid integration variable or limit(s) in 1. ≫

$\int 0 \, d1$

C[x]

C[x]

S[x]

S[x]

g[x] // FullSimplify

$\sqrt{\frac{\pi}{2}} \left(\text{Cos}[1]\,\text{FresnelS}\left[\sqrt{\frac{2}{\pi}}\ x\right] - \text{FresnelC}\left[\sqrt{\frac{2}{\pi}}\ x\right]\text{Sin}[1]\right)$

g[2]

Integrate::ilim : Invalid integration variable or limit(s) in 2. ≫

$\int \text{Sin}[3] \, d2$

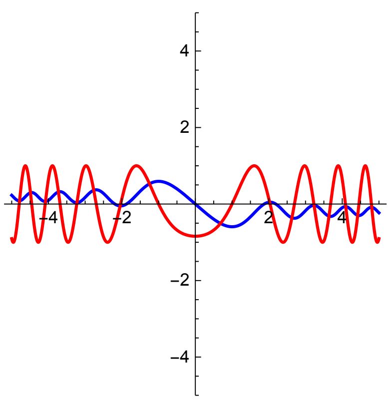
S[Sqrt[(2 / Pi) x]]

$\text{S}\left[\sqrt{\frac{2}{\pi}}\ \sqrt{x}\right]$

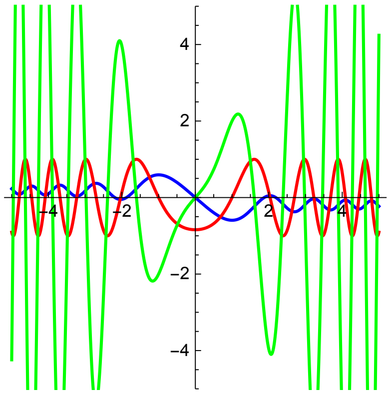
S[Sqrt[(2 / Pi) 2]]

$\text{S}\left[\frac{2}{\sqrt{\pi}}\right]$

ShowDerivatives[g, 2, -5, 5]



```
ShowDerivatives[g, 3, -5, 5]
```




```
h[t_] := Integrate[f[x], {x, 0, t}]
```

```
h[t]
```

$$\sqrt{\frac{\pi}{2}}\left(\text{Cos}[1]\ \text{FresnelS}\left[\sqrt{\frac{2}{\pi}}\ t\right]-\text{FresnelC}\left[\sqrt{\frac{2}{\pi}}\ t\right]\ \text{Sin}[1]\right)$$

```
h[3]
```

$$\sqrt{\frac{\pi}{2}}\left(\text{Cos}[1]\ \text{FresnelS}\left[3\sqrt{\frac{2}{\pi}}\right]-\text{FresnelC}\left[3\sqrt{\frac{2}{\pi}}\right]\ \text{Sin}[1]\right)$$

```
% // N
```

-0.173482

```
h[t] // N
```

1.25331 (-0.841471 FresnelC[0.797885 t] + 0.540302 FresnelS[0.797885 t])

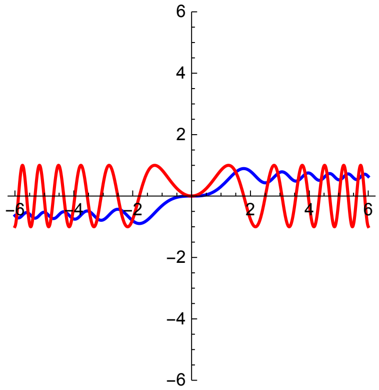
More Fresnel

```
Integrate[Sin[t^2], {t, 0, x}]
```

$$\sqrt{\frac{\pi}{2}}\ \text{FresnelS}\left[\sqrt{\frac{2}{\pi}}\ x\right]$$

```
g[x_] := Integrate[Sin[t^2], {t, 0, x}]
h[x_] := Sin[x^2]
```

```
ShowDerivatives[g, 2, -6, 6]
```



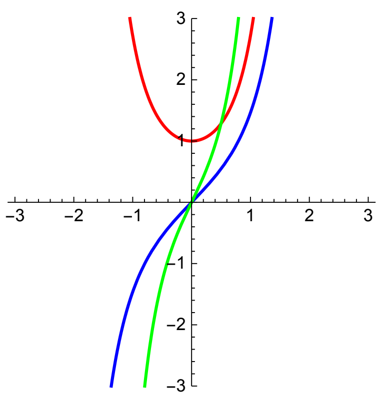
More exp

```
Integrate[Exp[t^2], {t, 0, x}]
```

$\frac{1}{2}\sqrt{\pi}\,\text{Erfi}[x]$

```
g[x_] := Integrate[Exp[t^2], {t, 0, x}]
h[x_] := Exp[x^2]
```

```
ShowDerivatives[g, 3, -3, 3]
```

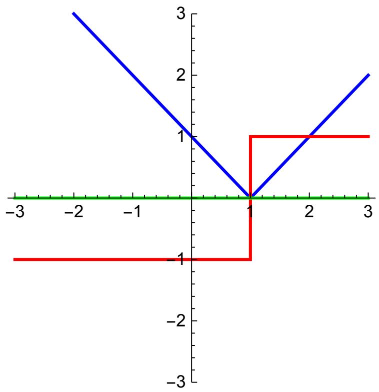



```
Remove[f]
```

Elliptic

```
f[x_] := Sqrt[(1 - x)^2]
```

```
ShowDerivatives[f, 3, -3, 3]
```

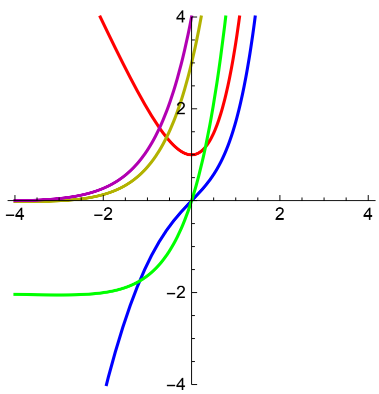


```
Remove[f]
```

More exp

```
f[x_] := x Exp[x] - x^2
```

```
ShowDerivatives[f, 5, -4, 4]
```



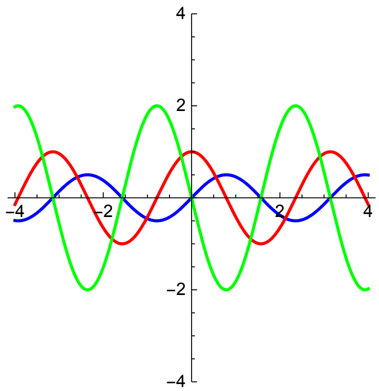
SinCos

```
f[x_] := Sin[x] Cos[x]
```

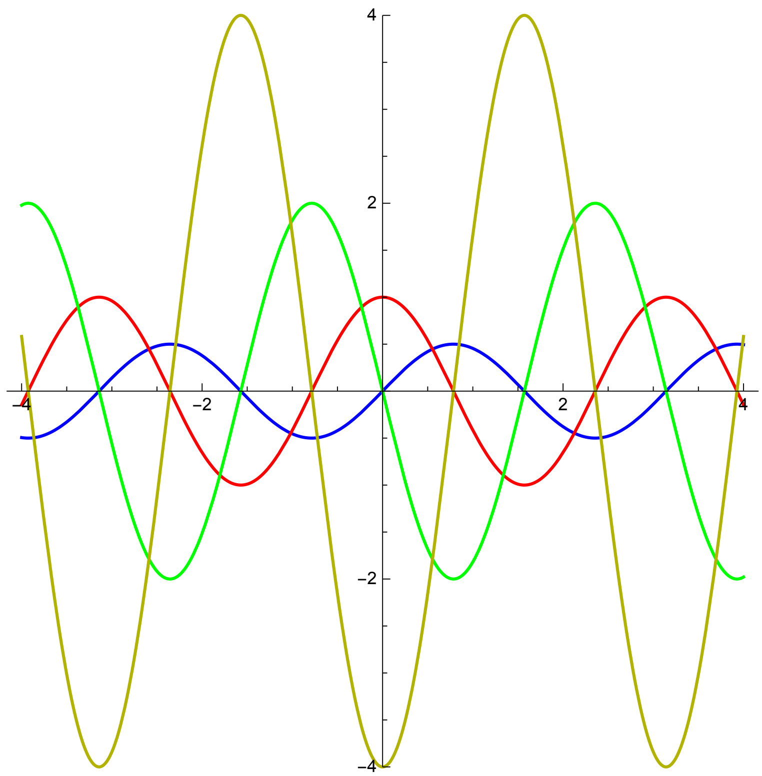
```
DF[f, 3] // TraditionalForm
```

function	$\cos(x)\sin(x)$
1st deriv	$\cos^2(x)-\sin^2(x)$
2nd deriv	$-4\cos(x)\sin(x)$

```
ShowDerivatives[f, 3, -4, 4]
```



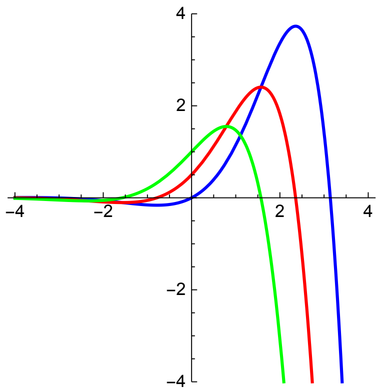

```
SD[f, 4, -4, 4, -4, 4, 400]
```



ExpSin

```
f[x_] := Exp[x] Sin[x] / 2
```

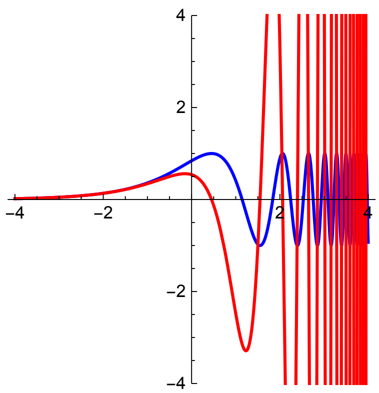
```
ShowDerivatives[f, 3, -4, 4]
```



SinExp

```
f[x_] := Sin[Exp[x]]
```

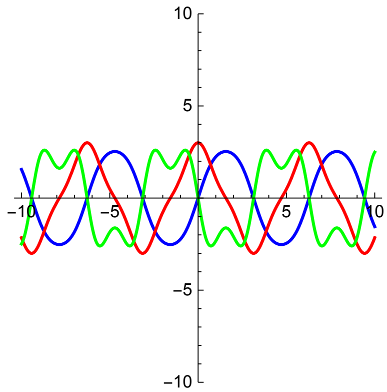
```
ShowDerivatives[f, 2, -4, 4]
```



SinSIn

```
f[x_] := 3 Sin[Sin[x]]
```

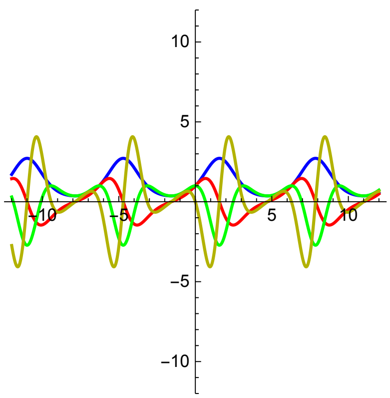
```
ShowDerivatives[f, 3, -10, 10]
```



expSin again

```
f[x_] := Exp[Sin[x]]
```

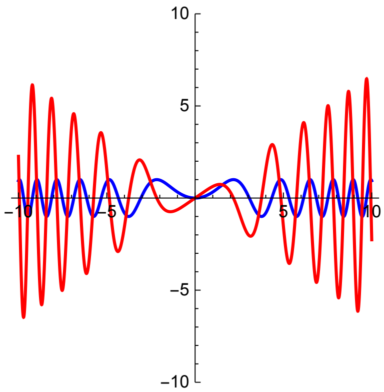
```
ShowDerivatives[f, 4, -12, 12]
```



More sin

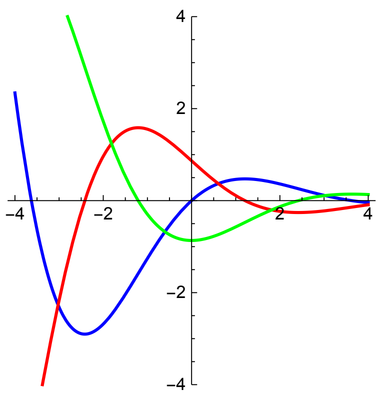
$$f[x\_] := \mathrm{Sin}\left[x^2 / 3\right]$$

```
ShowDerivatives[f, 2, -10, 10]
```



More exp sin

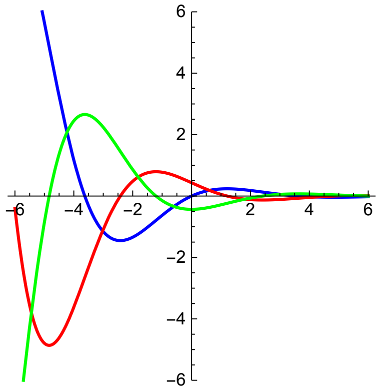
```
f[x_] := Exp[-x/2] Sin[Sqrt[3] x / 2]
ShowDerivatives[f, 3, -4, 4]
```




```
f[x_] := 1/2 Exp[-x/2] Sin[Sqrt[3] x / 2]
{f[x], f'[x], f''[x]}
```

$$\left\{\frac{1}{2} e^{-x/2} \operatorname{Sin}\left[\frac{\sqrt{3}\ x}{2}\right], \frac{1}{4}\sqrt{3}\ e^{-x/2} \operatorname{Cos}\left[\frac{\sqrt{3}\ x}{2}\right] - \frac{1}{4} e^{-x/2} \operatorname{Sin}\left[\frac{\sqrt{3}\ x}{2}\right], -\frac{1}{4}\sqrt{3}\ e^{-x/2} \operatorname{Cos}\left[\frac{\sqrt{3}\ x}{2}\right] - \frac{1}{4} e^{-x/2} \operatorname{Sin}\left[\frac{\sqrt{3}\ x}{2}\right]\right\}$$

```
ShowDerivatives[f, 3, -6, 6]
```

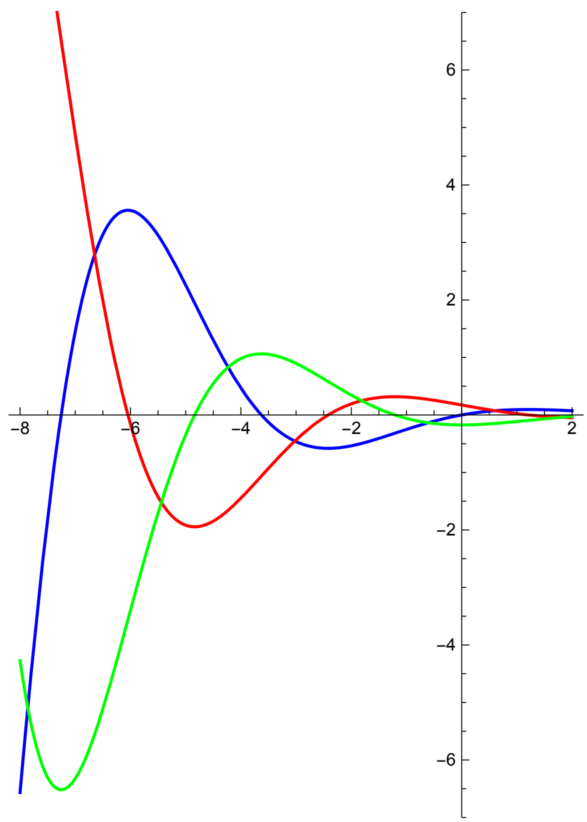



```
f[x_] := 1/5 Exp[-x/2] Sin[Sqrt[3] x / 2]
```

```
{f[x], f'[x], f''[x], f'''[x]}
```

$$\left\{\frac{1}{5} e^{-x/2} \operatorname{Sin}\left[\frac{\sqrt{3}\, x}{2}\right], \frac{1}{10} \sqrt{3}\, e^{-x/2} \operatorname{Cos}\left[\frac{\sqrt{3}\, x}{2}\right] - \frac{1}{10} e^{-x/2} \operatorname{Sin}\left[\frac{\sqrt{3}\, x}{2}\right], -\frac{1}{10} \sqrt{3}\, e^{-x/2} \operatorname{Cos}\left[\frac{\sqrt{3}\, x}{2}\right] - \frac{1}{10} e^{-x/2} \operatorname{Sin}\left[\frac{\sqrt{3}\, x}{2}\right], \frac{1}{5} e^{-x/2} \operatorname{Sin}\left[\frac{\sqrt{3}\, x}{2}\right]\right\}$$

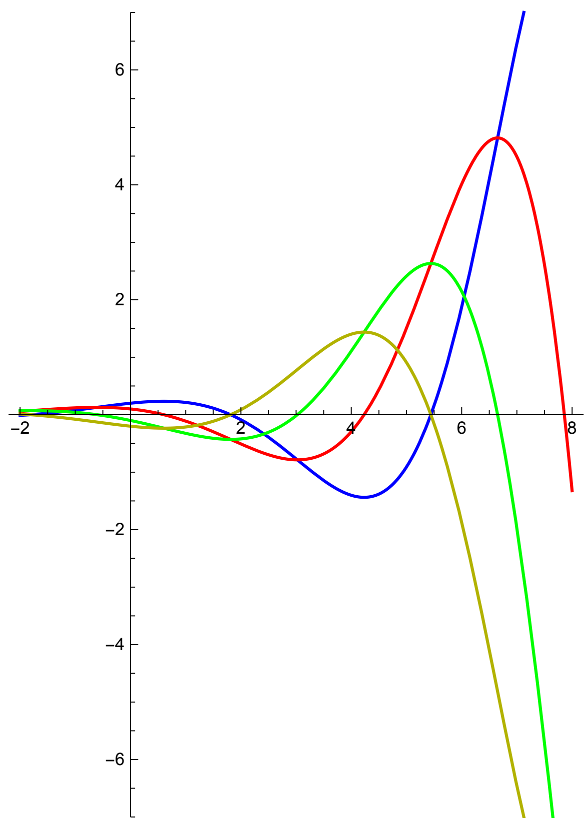
```
Plot[{f[x], f'[x], f''[x]}, {x, -8, 2}, PlotRange -> {-7, 7},
 AspectRatio → 7 / 5, Prolog → AbsoluteThickness[GT],
 PlotStyle -> {{RGBColor[0, 0, 1]}, {RGBColor[1, 0, 0]}, {RGBColor[0, 1, 0]},
   RGBColor[.7, .7, 0], RGBColor[0, .7, .6]}, ImageSize → {300, 600}]
```




```
f[x_] := 1/5 Exp[x/2] Cos[Sqrt[3] (- x) / 2]
{f[x], f'[x], f''[x], f'''[x]}
```

$$\left\{\frac{1}{5} e^{x/2} \operatorname{Cos}\left[\frac{\sqrt{3}\, x}{2}\right], \frac{1}{10} e^{x/2} \operatorname{Cos}\left[\frac{\sqrt{3}\, x}{2}\right] - \frac{1}{10} \sqrt{3}\, e^{x/2} \operatorname{Sin}\left[\frac{\sqrt{3}\, x}{2}\right], -\frac{1}{10} e^{x/2} \operatorname{Cos}\left[\frac{\sqrt{3}\, x}{2}\right] - \frac{1}{10} \sqrt{3}\, e^{x/2} \operatorname{Sin}\left[\frac{\sqrt{3}\, x}{2}\right], -\frac{1}{5} e^{x/2} \operatorname{Cos}\left[\frac{\sqrt{3}\, x}{2}\right]\right\}$$

```
Plot[{f[x], f'[x], f''[x], f'''[x]}, {x, -2, 8},
 PlotRange -> {-7, 7}, AspectRatio → 7 / 5, Prolog → AbsoluteThickness[GT],
 PlotStyle -> {{RGBColor[0, 0, 1]}, {RGBColor[1, 0, 0]}, {RGBColor[0, 1, 0]},
   RGBColor[.7, .7, 0], RGBColor[0, .7, .6]}, ImageSize → {300, 600}]
```




```
f[x_] := 1/5 Exp[x/2] Sin[Sqrt[3] (-x) / 2]
{f[x], f'[x], f''[x], f'''[x]}
```

$$\left\{-\frac{1}{5}\, e^{x/2} \,\text{Sin}\left[\frac{\sqrt{3}\, x}{2}\right],\ -\frac{1}{10}\sqrt{3}\, e^{x/2}\,\text{Cos}\left[\frac{\sqrt{3}\, x}{2}\right] - \frac{1}{10}\, e^{x/2}\,\text{Sin}\left[\frac{\sqrt{3}\, x}{2}\right],\right.$$

$$\left. -\frac{1}{10}\sqrt{3}\, e^{x/2}\,\text{Cos}\left[\frac{\sqrt{3}\, x}{2}\right] + \frac{1}{10}\, e^{x/2}\,\text{Sin}\left[\frac{\sqrt{3}\, x}{2}\right],\ \frac{1}{5}\, e^{x/2}\,\text{Sin}\left[\frac{\sqrt{3}\, x}{2}\right]\right\}$$

```
f[x_] := 1/5 Exp[x/2] Sin[Sqrt[3] (x) / 2]
```

```
{f[x], f'[x], f''[x], f'''[x]}
```

$$\left\{\frac{1}{5} e^{x/2} \text{Sin}\left[\frac{\sqrt{3}\ x}{2}\right], \frac{1}{10}\sqrt{3}\ e^{x/2} \text{Cos}\left[\frac{\sqrt{3}\ x}{2}\right] + \frac{1}{10} e^{x/2} \text{Sin}\left[\frac{\sqrt{3}\ x}{2}\right], \frac{1}{10}\sqrt{3}\ e^{x/2} \text{Cos}\left[\frac{\sqrt{3}\ x}{2}\right] - \frac{1}{10} e^{x/2} \text{Sin}\left[\frac{\sqrt{3}\ x}{2}\right], -\frac{1}{5} e^{x/2} \text{Sin}\left[\frac{\sqrt{3}\ x}{2}\right]\right\}$$

```
f[x_] := 1/5 Exp[-x/2] Sin[Sqrt[3] (x) / 2]
```

```
{f[x], f'[x], f''[x], f'''[x]}
```

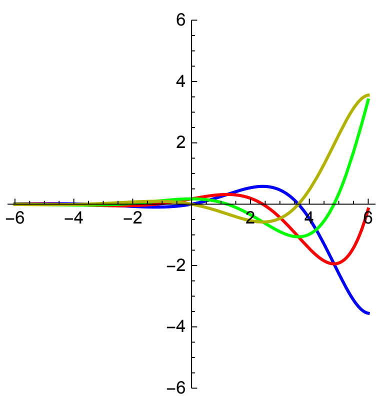
$$\left\{\frac{1}{5} e^{-x/2} \text{Sin}\left[\frac{\sqrt{3}\ x}{2}\right], \frac{1}{10}\sqrt{3}\ e^{-x/2} \text{Cos}\left[\frac{\sqrt{3}\ x}{2}\right] - \frac{1}{10} e^{-x/2} \text{Sin}\left[\frac{\sqrt{3}\ x}{2}\right], -\frac{1}{10}\sqrt{3}\ e^{-x/2} \text{Cos}\left[\frac{\sqrt{3}\ x}{2}\right] - \frac{1}{10} e^{-x/2} \text{Sin}\left[\frac{\sqrt{3}\ x}{2}\right], \frac{1}{5} e^{-x/2} \text{Sin}\left[\frac{\sqrt{3}\ x}{2}\right]\right\}$$

```
f[x_] := -1/5 Exp[x/2] Sin[Sqrt[3] (-x) / 2]
```

```
{f[x], f'[x], f''[x], f'''[x]}
```

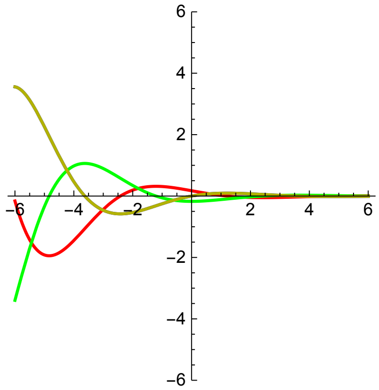
$$\left\{\frac{1}{5} e^{x/2} \text{Sin}\left[\frac{\sqrt{3}\ x}{2}\right], \frac{1}{10}\sqrt{3}\ e^{x/2} \text{Cos}\left[\frac{\sqrt{3}\ x}{2}\right] + \frac{1}{10} e^{x/2} \text{Sin}\left[\frac{\sqrt{3}\ x}{2}\right], \frac{1}{10}\sqrt{3}\ e^{x/2} \text{Cos}\left[\frac{\sqrt{3}\ x}{2}\right] - \frac{1}{10} e^{x/2} \text{Sin}\left[\frac{\sqrt{3}\ x}{2}\right], -\frac{1}{5} e^{x/2} \text{Sin}\left[\frac{\sqrt{3}\ x}{2}\right]\right\}$$

```
ShowDerivatives[f, 4, -6, 6]
```

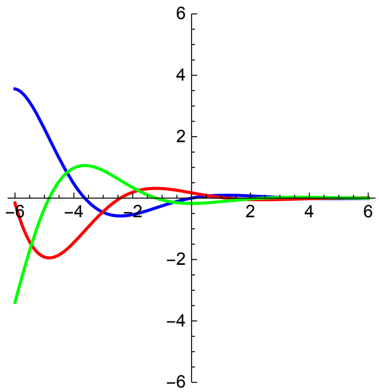



```
f[x_] := 1/5 Exp[-x/2] Sin[Sqrt[3] (x) / 2]
```

```
ShowDerivatives[f, 4, -6, 6]
```

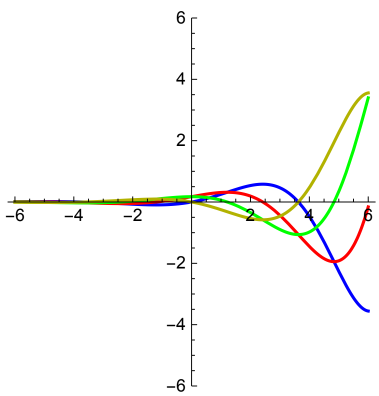



```
ShowDerivatives[f, 3, -6, 6]
```



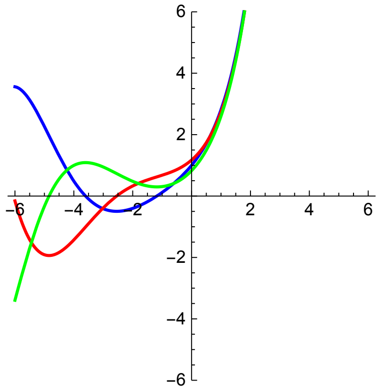

```
f[x_] := 1/5 Exp[-x/2] Sin[Sqrt[3] (x) / 2] Exp[x]
```

```
ShowDerivatives[f, 4, -6, 6]
```



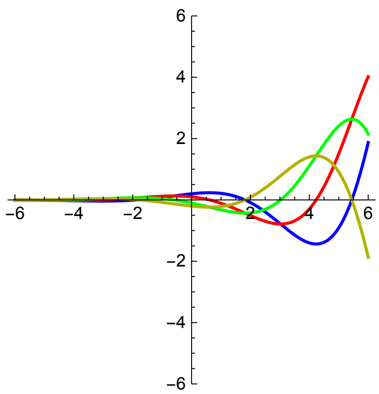

```
f[x_] := 1/5 Exp[-x/2] Sin[Sqrt[3] (x) / 2] + Exp[x]
```

```
ShowDerivatives[f, 3, -6, 6]
```




```
f[x_] := 1/5 Exp[x/2] Cos[Sqrt[3] (- x ) / 2]
```

```
ShowDerivatives[f, 4, -6, 6]
```




```
f[x_] := 1/2 Exp[-x/2] Sin[a x]
```

```
{f[x], f'[x], f''[x], f'''[x]}
```

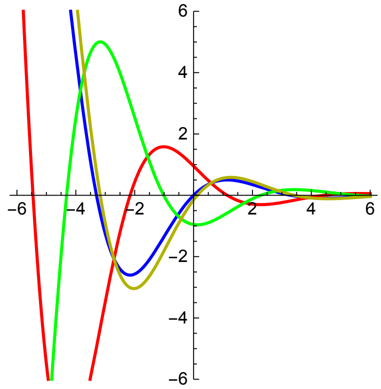
$$\left\{\frac{1}{2} e^{-x/2} \text{Sin}[a\,x],\ \frac{1}{2} a\, e^{-x/2} \text{Cos}[a\,x] - \frac{1}{4} e^{-x/2} \text{Sin}[a\,x],\right.$$
$$-\frac{1}{2} a\, e^{-x/2} \text{Cos}[a\,x] + \frac{1}{8} e^{-x/2} \text{Sin}[a\,x] - \frac{1}{2} a^2 e^{-x/2} \text{Sin}[a\,x],$$
$$\left.\frac{3}{8} a\, e^{-x/2} \text{Cos}[a\,x] - \frac{1}{2} a^3 e^{-x/2} \text{Cos}[a\,x] - \frac{1}{16} e^{-x/2} \text{Sin}[a\,x] + \frac{3}{4} a^2 e^{-x/2} \text{Sin}[a\,x]\right\}$$

```
f[x_] := Exp[-x/2] Sin[1.1 Sqrt[3] x / 2]
```

```
DF[f, 5]
```

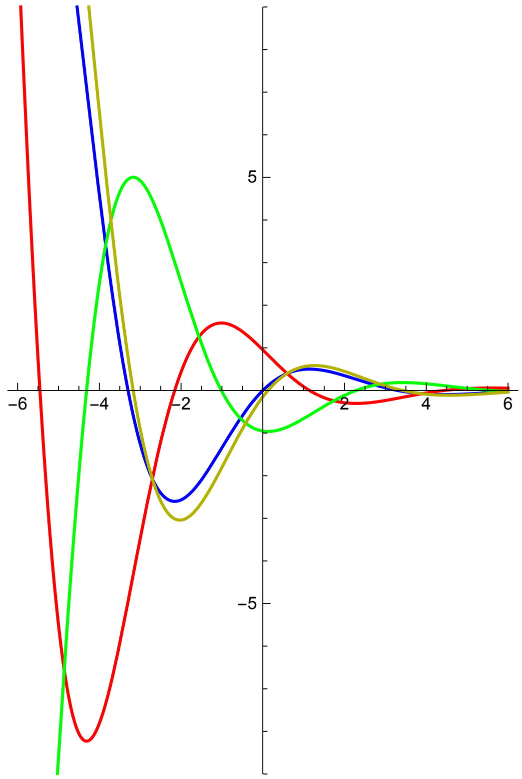
function	$e^{-x/2} \sin(0.952628\, x)$
1st deriv	$0.952628\, e^{-x/2} \cos(0.952628\, x) - \frac{1}{2} e^{-x/2} \sin(0.952628\, x)$
2nd deriv	$-0.6575\, e^{-x/2} \sin(0.952628\, x) - 0.952628\, e^{-x/2} \cos(0.952628\, x)$
3rd deriv	$1.23625\, e^{-x/2} \sin(0.952628\, x) - 0.150039\, e^{-x/2} \cos(0.952628\, x)$
4th deriv	$1.25271\, e^{-x/2} \cos(0.952628\, x) - 0.475194\, e^{-x/2} \sin(0.952628\, x)$

ShowDerivatives[f, 4, -6, 6]



SD[f, 5, -6, 6, -10, 6, 400]

```
Plot[{f[x], f'[x], f''[x], f'''[x]}, {x, -6, 6},
 PlotRange -> {-9, 9}, AspectRatio → 3/2, Prolog → AbsoluteThickness[GT],
 PlotStyle -> {{RGBColor[0, 0, 1]}, {RGBColor[1, 0, 0]}, {RGBColor[0, 1, 0]},
   RGBColor[.7, .7, 0], RGBColor[0, .7, .6]}, ImageSize → {600, 400}]
```

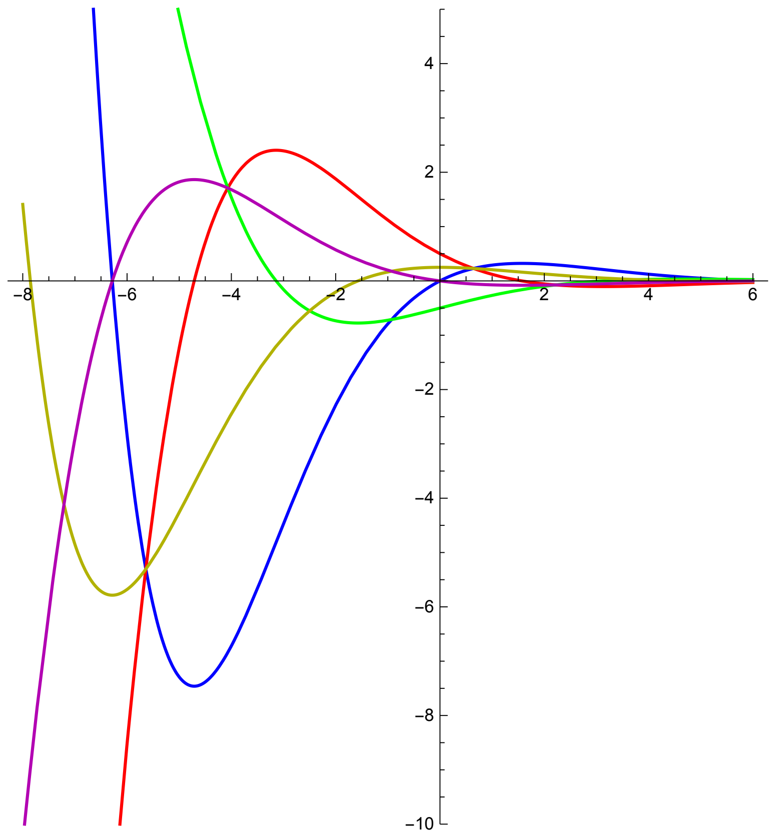



```
f[x_] := Exp[-x/2] Sin[ x / 2]
```

```
DF[f, 5]
```

function	$e^{-x/2} \sin\left(\frac{x}{2}\right)$
1st deriv	$\frac{1}{2} e^{-x/2} \cos\left(\frac{x}{2}\right) - \frac{1}{2} e^{-x/2} \sin\left(\frac{x}{2}\right)$
2nd deriv	$-\frac{1}{2} e^{-x/2} \cos\left(\frac{x}{2}\right)$
3rd deriv	$\frac{1}{4} e^{-x/2} \sin\left(\frac{x}{2}\right) + \frac{1}{4} e^{-x/2} \cos\left(\frac{x}{2}\right)$
4th deriv	$-\frac{1}{4} e^{-x/2} \sin\left(\frac{x}{2}\right)$

```
SD[f, 5, -8, 6, -10, 5, 400]
```



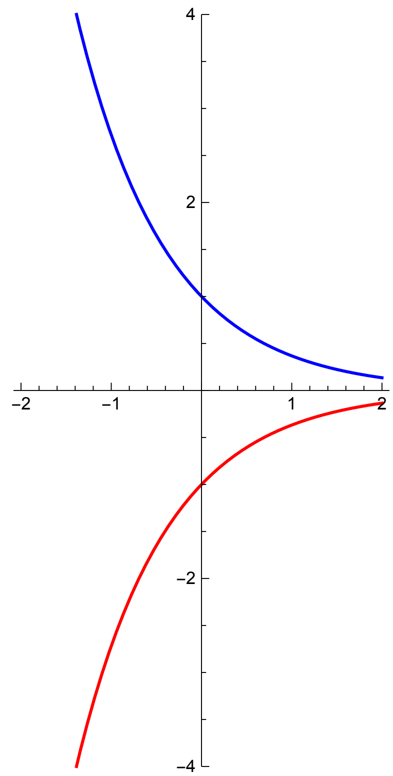

```
Remove[f]
```

```
f[x_] := Sin[1.1 x]
```

More exp

```
f[x_] := Exp[-x]
```

```
Plot[{f[x], f'[x]}, {x, -2, 2}, PlotRange -> {-4, 4},
 AspectRatio → 2, Prolog → AbsoluteThickness[GT],
 PlotStyle -> {{RGBColor[0, 0, 1]}, {RGBColor[1, 0, 0]}, {RGBColor[0, 1, 0]},
   RGBColor[.7, .7, 0], RGBColor[0, .7, .6]}, ImageSize → {400, 400}]
```



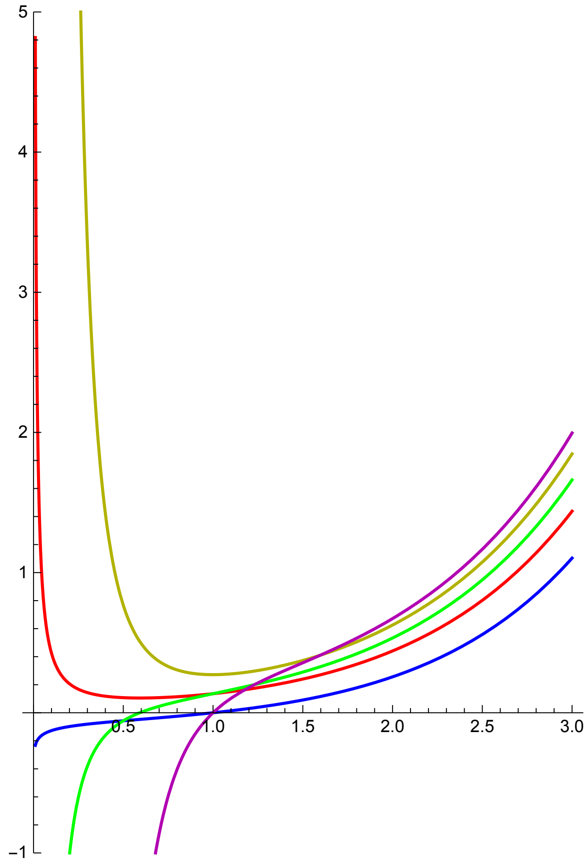
Log times exponential

```
f[x_] := .05 Log[x] Exp[x]
```

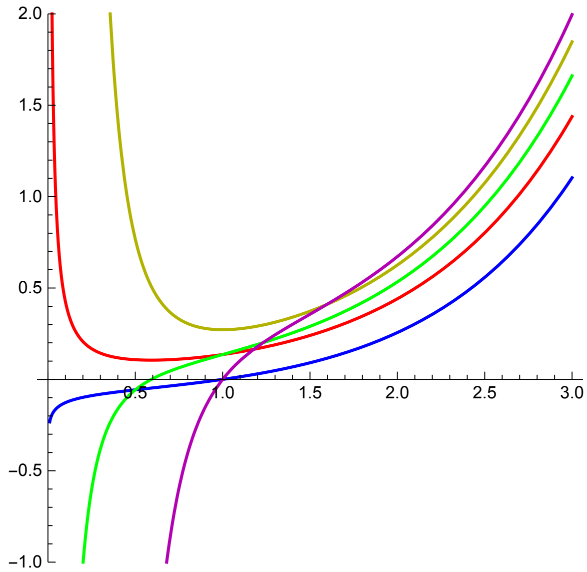
```
Plot[{f[x], f'[x], f''[x], f'''[x], f''''[x]}, {x, .01, 3},
 PlotRange -> {-1, 5}, AspectRatio → 3/2, Prolog → AbsoluteThickness[GT],
 PlotStyle -> {{RGBColor[0, 0, 1]}, {RGBColor[1, 0, 0]}, {RGBColor[0, 1, 0]},
   RGBColor[.7, .7, 0], RGBColor[0 = .7, 0, .7]}, ImageSize → {300, 450}]
```

Set::setraw : Cannot assign to raw object 0. ≫

Set::setraw : Cannot assign to raw object 0. ≫



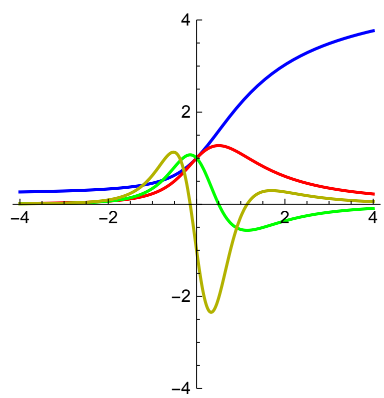
```
SD[f, 5, 0.01, 3, -1, 2, 300]
```



exp and arctan

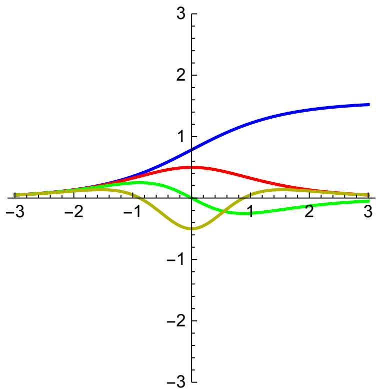
```
f[x_] := Exp[ArcTan[x]]
```

```
ShowDerivatives[f, 4, -4, 4]
```




```
f[x_] := ArcTan[Exp[x]]
```

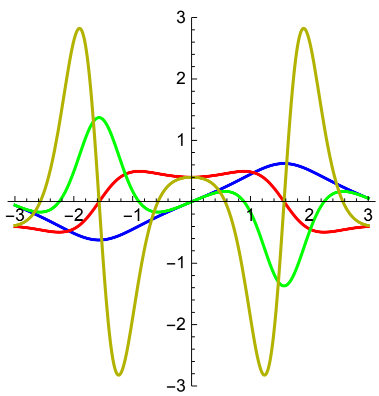
```
ShowDerivatives[f, 4, -3, 3]
```



tan and sin

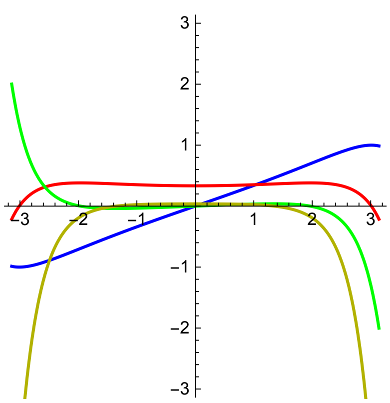
```
f[x_] := .4 Tan[Sin[x]]
```

```
ShowDerivatives[f, 4, -3, 3]
```

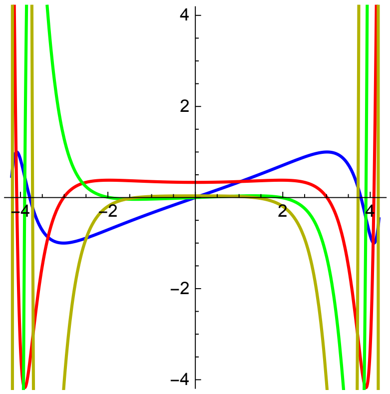



```
f[x_] := Sin[Tan[x / 3]]
```

```
ShowDerivatives[f, 4, -Pi, Pi]
```



```
ShowDerivatives[f, 4, -4.2, 4.2]
```



Exponential times Sine

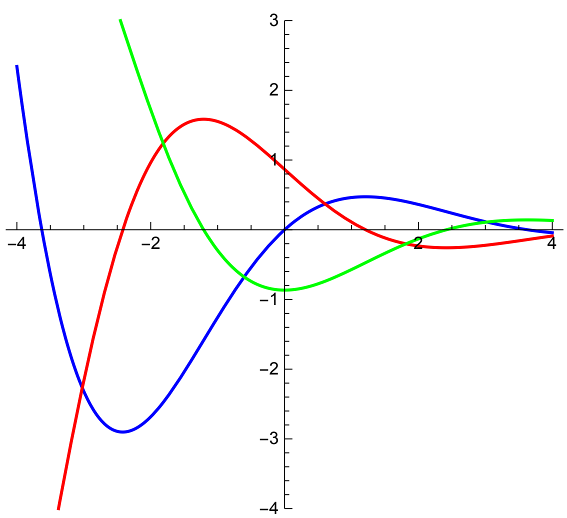
```
Remove[f]
```

```
DSolve[y'''[x] == y[x], y, x]
```

$$\left\{\left\{y \to \text{Function}\left[\{x\}, e^{x}\,C[1] + e^{-x/2}\,C[2]\,\text{Cos}\left[\frac{\sqrt{3}\,x}{2}\right] + e^{-x/2}\,C[3]\,\text{Sin}\left[\frac{\sqrt{3}\,x}{2}\right]\right]\right\}\right\}$$

```
f[x_] := Exp[-x/2] Sin[(Sqrt[3]/2) x]
```

```
SD[f, 3, -4, 4, -4, 3, 300]
```



Sine of a quadratic

```
squad[x_] := Sin[x^2 - 1]
```

```
DF[squad, 5]
```

function	$-\sin(1-x^2)$
1st deriv	$2x\cos(1-x^2)$
2nd deriv	$4x^2\sin(1-x^2)+2\cos(1-x^2)$
3rd deriv	$12x\sin(1-x^2)-8x^3\cos(1-x^2)$
4th deriv	$12\sin(1-x^2)-48x^2\cos(1-x^2)-16x^4\sin(1-x^2)$

Note that SD prints the graph of the derivative *after* the graph of the function so that the blue seems to disappear toward the left and the right. This is an artefact and needs to be fixed.

```
SD[squad, 2, -12, 12, -12, 12, 300]
```